hw set 18
sect 6.5
p 479

2. distance = x , Force = $\cos(\pi x/3)$
work from $x=1$ to $x=2$?
work [1,1.5] & [1.5,2] ?

$$W = \int_1^2 \cos(\pi x/3)\,dx$$
$$= \tfrac{3}{\pi}\sin(\pi x/3)\Big|_1^2$$
$$= \frac{3}{\pi}\left[\frac{\sqrt3}{2} - \frac{\sqrt3}{2}\right] = 0$$
$$W_1 = \int_1^{1.5} \cos(\pi x/3)\,dx$$
$$= \frac{3}{\pi}\left[1 - \frac{\sqrt3}{2}\right] = \frac{-3\sqrt3+6}{2\pi}$$
$$W_2 = \int_{1.5}^{2} \cos(\pi/3\, x)\,dx$$
$$= \frac{3}{\pi}\left[\frac{\sqrt3}{2} - 1\right] = \frac{3\sqrt3-6}{2\pi}$$

Thus, no total work as we found.

3. distance = 8 m
force funct (graph)

$$W = \int_0^8 F(x)\,dx$$
$$= \tfrac{1}{2}(4\cdot 30) + 120 = 180$$

5. F = 10 lb on spring stretched 4 in past natural length. Find work when stretching spring from natural length to 6 in beyond natural length.

Hook's Law: $f(x) = kx$

US units: F = lb, d = ft, W = ft-lb
so 4 in = $\frac{1}{3}$ ft & 6 in = $\frac{1}{2}$ ft

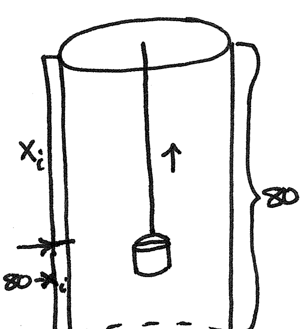
to find k:

$$f(\tfrac{1}{3}-0) = k(\tfrac{1}{3}-0) = 10$$
$$\tfrac{1}{3}k = 10,\quad k = \cancel{2.5} \; 30$$

$f(x) = 30x$ where x = distance

$$W = \int_0^{.5} 30x\,dx = 15x^2\Big|_0^{1/2} = \frac{15}{4}$$

12. (similar to Example 3)
bucket weighs 4 lb, well 80 ft
bucket holds 40 lb water
pulled rate = 2 ft/s, leaks rate = .2 ft/s



use D = RT

distance = x_i, [0, 80]

Time at x_i:
$D = 80 - x_i$, $R = 2$
$T = \frac{80-x_i}{2}$ sec

Leak at x_i:
$$(.2\,\text{lb/s})\left(\frac{80-x_i}{2}\,\text{s}\right) = 8 - \frac{x_i}{10}\ \text{lb}$$

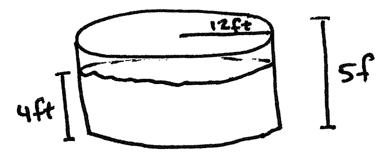
weight at x_i:
initial weight = 4 + 40 = 44 lb
− leak = $-(8 - \frac{x_i}{10})$ lb
weight = $44 - 8 + \frac{x_i}{10} = 36 + \frac{x_i}{10}$ lb

Now since vertical: Weight = Force

Work at x_i = $x_i(36 + \frac{x_i}{10})$

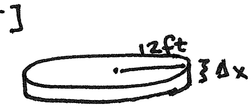
$$W = \int_0^{80} 36x + \tfrac{1}{10}x^2\,dx = \frac{396800}{3} = 132266.\overline{6}$$

16. Find work to pump H_2O over side. (H_2O weighs 62.5 lb/ft³)



− water goes from 1 ft below top to 5 ft below
distance = x , [1, 5]

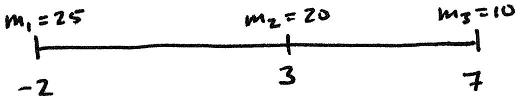
− Layer:



$$V_i = r_i^2\pi\,\Delta x$$
$$= 12^2\pi\,\Delta x = 144\pi\,\Delta x\ \text{ft}^3$$
weight = $62.5 \cdot 144\pi\,\Delta x$ lb = $9000\pi\,\Delta x$ lb
$W_i = 9000\pi x_i\,\Delta x$ lb

$$W = \int_1^5 9000\pi x\,dx = 4500\pi x^2\Big|_1^5$$
$$= 108000\pi = 339292.0066$$

32. Find the moment M
& the center of mass $\bar{x}$

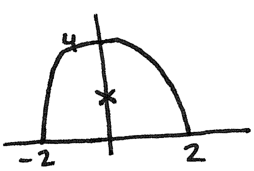


$M = (-2)(25) + 3(20) + 7(10) = 80$

$m = 25 + 20 + 10 = 55$

$\bar{x} = 80/55 = 16/11 = 1.45$

35. $y = 4 - x^2$, $y = 0$



$A = \int_{-2}^{2} 4 - x^2\,dx$

$= 2\int_0^2 4 - x^2\,dx = \frac{32}{3}$

$\bar{x} = \frac{1}{A}\int_a^b x f(x)\,dx = \frac{3}{32}\int_{-2}^{2} x(4-x^2)\,dx$

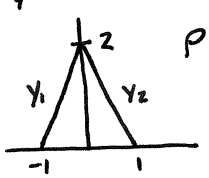
$= \frac{3}{32}\int_{-2}^{2} 4x - x^3\,dx = 0$

$\bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2\,dx = \frac{3}{64}\int_{-2}^{2} (4-x^2)^2\,dx = 8/5$

centroid $= (0, 8/5)$

39. Find M_x & M_y & centroid

$\begin{cases} y_1 = 2x+2 \\ y_2 = -2x+2 \end{cases}$



$\rho = 1$

$A = \frac{1}{2}(2)(2) = 2$

$M_y = \rho\int_a^b x f(x)\,dx$

$= \int_{-1}^{0} (2x+2)x\,dx + \int_0^1 x(-2x+2)\,dx$

$= \int_{-1}^{0} 2x^2 + 2x\,dx + \int_0^1 -2x^2 + 2x\,dx$

$= -1/3 + 1/3 = 0$

$M_x = \rho\int_a^b \frac{1}{2}[f(x)]^2\,dx$

$= \int_{-1}^{0} \frac{1}{2}(2x+2)^2\,dx + \int_0^1 \frac{1}{2}(-2x+2)^2\,dx$

$= 2/3 + 2/3 = 4/3$

$\bar{x} = \frac{M_y}{m} = \frac{0}{\rho A} = \frac{0}{2} = 0$

$\bar{y} = \frac{M_x}{m} = \frac{(4/3)}{\rho A} = \frac{(4/3)}{2} = 2/3$

centroid $= (0, 2/3)$

34. $m_1 = 6$, $m_2 = 5$, $m_3 = 1$, $m_4 = 4$

$P_1(1,-2)$, $P_2(3,4)$, $P_3(-3,-7)$, $P_4(6,-1)$

Find M_x, M_y, & center of mass

$m = 6 + 5 + 1 + 4 = 16$

$M_x = 6(-2) + 5(4) + 1(-7) + 4(-1) = -3$

$M_y = 6(1) + 5(3) + 1(-3) + 4(6) = 42$

$\bar{x} = \frac{M_y}{m} = \frac{42}{16} = \frac{21}{8} = 2.625$

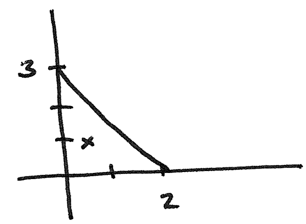
$\bar{y} = \frac{M_x}{m} = \frac{-3}{16} = -.1875$

center of mass $= (\bar{x}, \bar{y}) = \left(\frac{21}{8}, \frac{-3}{16}\right)$

36. $3x + 2y = 6$, $y = 0$, $x = 0$

$2y = 6 - 3x$

$y = 3 - \frac{3}{2}x$



$A = \frac{1}{2}\cdot 2\cdot 3 = 3$

$\bar{x} = \frac{1}{A}\int_a^b x f(x)\,dx$

$= \frac{1}{3}\int_0^2 x\left(3 - \frac{3}{2}x\right)dx$

$= \frac{1}{6}\int_0^2 6x - 3x^2\,dx$

$= \frac{1}{6}\left[3x^2 - x^3\right]_0^2 = \frac{1}{6}(12-8) = \frac{2}{3}$

$\bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2\,dx$

$= \frac{1}{6}\int_0^2 \left(3 - 3/2\,x\right)^2\,dx = 1$

centroid $= (2/3, 1)$